

# Exercise V

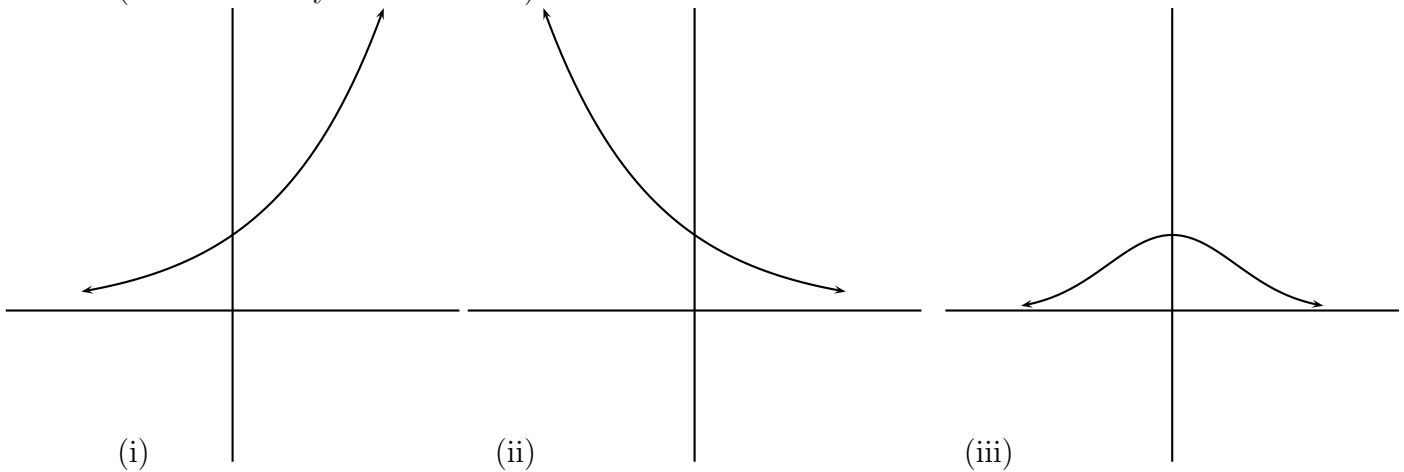
- How do we know that the infinite decimal

0.112123123412345123456123456712345678123456789123456789101234567891011...

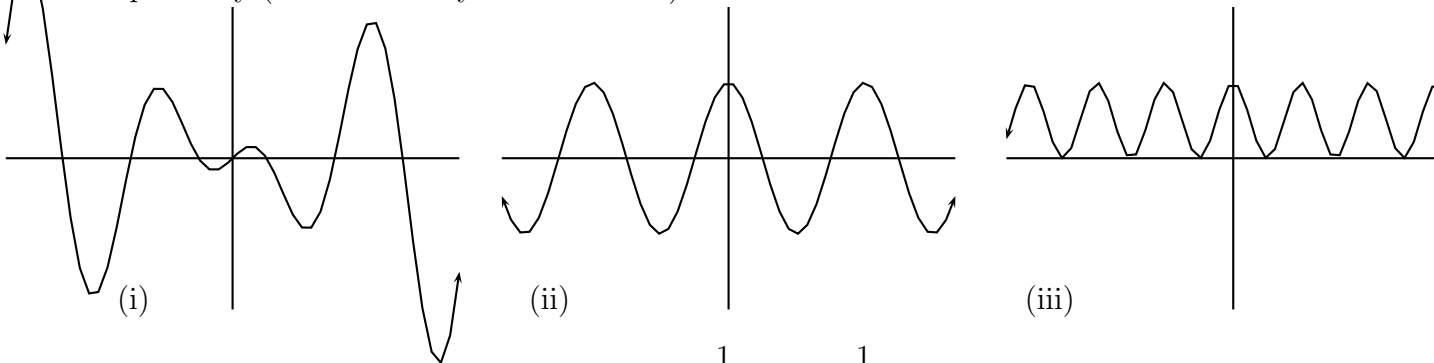
converges to an irrational number?

- How do we know that the series  $\sum_{k=1}^{\infty} \frac{1}{10^{2^k}}$  converges to an irrational number?

- The following diagrams show sketches of the functions  $2^{-x^2}$ ,  $2^x$  and  $2^{-x}$  respectively (not necessarily in that order.) Determine which is which.



- The following diagrams show sketches of the functions  $\cos(x)$ ,  $\cos^2(x)$  and  $x \cos(x)$  respectively (not necessarily in that order.) Determine which is which.



- Find a rational number that lies between  $\frac{1}{1000}$  and  $\frac{1}{1001}$ .

- Find a rational number that lies between  $\sqrt{2}$  and  $\sqrt{3}$ .

- Find an irrational number that lies between  $\sqrt{2}$  and  $\sqrt{3}$ . You can express your answer as an infinite decimal.

(Hint:  $\sqrt{2} = 1.4\dots$ ,  $\sqrt{3} = 1.7\dots$ )